## Determination of Thermodynamic Properties of Boehmite from its Solubility Data in NaOH Solutions<sup>1)</sup>

Byong-Tae Chang

Department of Chemistry, Faculty of Science, Korea University, Kodaira, Tokyo 187 (Received February 13, 1981)

Thermodynamic properties of boehmite in alkaline pH range are determined by applying the extended Debye-Hückel theory to its solubility data in NaOH solutions. When  $\log(K_s/a_\pi^2)$ , where  $K_s = m_{\rm H} m_{\rm A1}$  and  $a_{\rm w}$  is the activity of the water in NaOH solutions, is plotted against a Debye-Hückel function of ionic strength(I),  $I^{1/2}/(1+B_t a_1 I^{1/2})$ , at a given temperature, the curve approaches gradually the theoretical Debye-Hückel limiting slope( $2A_t$ ) with decrease in the ionic strength, and at high ionic strength it deviates positively from the straight line. Under the assumption of the same ionic size parameters for OH<sup>-</sup> and Al(OH)<sub>4</sub><sup>-</sup> ions, the ionic solubility product,  $K_s^* = a_{\rm H} a_{\rm A1}/a_s^*$ , for boehmite is given by following equations in the temperature range from 80 to 250 °C:  $\log K_s^* = -2663/T - 5.71$  (Case 1) or  $\log K_s^* = -2764/T - 5.45$  (Case 2). From a comparison of the temperature dependences of  $K_s^6$  for boehmite and gibbsite, a transition temperature between the two alumina hydrates was calculated to be  $60\pm3$  °C.

Thermodynamics of Al<sub>2</sub>O<sub>3</sub>–Na<sub>2</sub>O–H<sub>2</sub>O system is very important not only for alumina production by the Bayer process, but also for understanding corrosion and passivation of aluminium in water and aqueous geochemistry of aluminous minerals.<sup>2–8</sup>) In the previous paper,<sup>1</sup>) based on the Debye-Hückel theory, a more reasonable method was presented in order to determine thermodynamic properties of alumina hydrates from their solubility data, and was applied to the solubility data of gibbsite, one of alumina trihydrates, in NaOH solutions with success.

The purpose of this paper is to evaluate several thermodynamic properties of boehmite, one of alumina monohydrates, by applying the similar procedure to solubility data of boehmite in NaOH solutions at temperatures up to 300 °C.<sup>2,9)</sup> This problem may be also interesting as a study on the thermodynamics and the Debye-Hückel theory of high temperature aqueous solutions.<sup>10,11)</sup>

## Theoretical

One can express a dissolution process of alumina monohydrate and the dissociation of water as follows:

$$AlOOH(s) + 2H_2O(1) = Al(OH)_4^-(1) + H^+(1),$$
 (1)

$$K_s^{\circ} = a_{\rm Al} a_{\rm H} / a_{\rm w}^2 = K_s \gamma_1^2 / a_{\rm w}^2,$$
 (2)

$$H_2O(1) = H^+(1) + OH^-(1),$$
 (3)

$$K_{\mathbf{w}}^{\circ} = a_{\mathbf{H}} a_{\mathbf{OH}} / a_{\mathbf{w}} = K_{\mathbf{w}} \gamma_{2}^{2} / a_{\mathbf{w}}, \tag{4}$$

where  $K_s = m_{A1} m_H$ ,  $K_w = m_H m_{OH}$ ,  $m_1$  the molality of species i,  $\gamma_1^2 = \gamma_{A1} \gamma_H$ ,  $\gamma_2^2 = \gamma_H \gamma_{OH}$ , and  $a_w$  the activity of the water in the molar fraction unit. From Eqs. 2 and 4 we can derive a general equation for the alumina monohydrate, introducing a new function, f(I), of ionic strength I:

$$f(I) = \log (m_{AI}/m_{OH}) + \log (K_{w}^{\circ}/a_{w})$$
  
= \log K\_{\sigma}^{\circ} + 2 \log (\gamma\_{2}/\gamma\_{1}). (5)

It should be noted that, in the case of the monohydrate, f(I) involves a term of  $a_{\rm w}$ , differing from the case of the trihydrate. Evaluation of the  $a_{\rm w}$  values at a given temperature and pressure of sodium aluminate and hydroxide solutions, therefore, become much important together with the solubility data. Russell et al. have used the  $a_{\rm w}$  values of corresponding NaOH

solutions at 25 °C for the sodium aluminate and hydroxide solutions up to 170 °C without any correction.<sup>2)</sup> Dibrov *et al.* studied in detail the saturated vapor pressure exerted by the sodium aluminate and hydroxide solutions in the wide range of temperatures and concentrations.<sup>12)</sup> In this paper, the  $a_{\rm w}$  values at a given temperature and concentration were calculated by using the vapor pressure data by Dibrov and virial coefficients, B' and C' of  $H_2O(g)$  by Kell,<sup>13)</sup> that is,

$$a_{\mathbf{w}} = f_{\mathbf{i}}/f_{\mathbf{o}} = \gamma_{\mathbf{i}}^{\mathbf{v}} P_{\mathbf{i}}/\gamma_{\mathbf{o}}^{\mathbf{v}} P_{\mathbf{o}}, \tag{6}$$

$$RT \ln (f_i/P_i) = RT \ln \gamma_i^* = B'P_i + C'P_i^2,$$
 (7)

where  $f_1$  and  $\gamma^{\gamma}$  are the fugacity and activity coefficient of  $H_2O(g)$ , respectively, and a subscript "o" means the pure water.

Figure 1 gives the relationships between f(I) and I obtained from Eq. 5, using the solubility data of boehmite reported by Russell  $(80-170 \,^{\circ}\text{C})^{2)}$  and Bernshtein  $(250 \, \text{and} \, 300 \,^{\circ}\text{C})^{9)}$  and the  $a_{\text{w}}$  values calculated by Eqs. 6 and 7. The  $K_{\text{w}}^{\circ}$  values were quoted from Refs. 14 and 15.

On the other hand, using the following extended

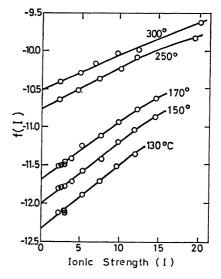


Fig. 1. Relationships between f(I) and ionic strength (I) according to Eq. 5 at temperatures from 130 to 300 °C.

Debye-Hückel equation,

$$\log \gamma_{\rm i} = -A_{\rm t} I^{1/2}/(1 + B_{\rm t} a_{\rm i} I^{1/2}) - c_{\rm i} I - d_{\rm i} I^2, \tag{8}$$

the function, f(I), can be expressed by

$$f(I) = \log K_s^{\circ} + 2A_t I^{1/2} [1/(1 + B_t a_1 I^{1/2}) - 1/(1 + B_t a_2 I^{1/2})] + CI + DI^2,$$
(9)

where  $A_{\rm t}$  is the theoretical Debye-Hückel limiting slope, and  $B_{\rm t}$ ,  $a_{\rm l}$ ,  $c_{\rm l}$ ,  $d_{\rm l}$ ,  $C=2(c_1-c_2)$  and  $D=2(d_1-d_2)$  are adjustable parameters. The problem is to decide the five parameters in Eq. 9 to get the best fit for the data given in Fig. 1.

## Results and Discussion

The Case of the Same Ionic Size Parameters. Setting up the same ionic size parameters, that is,  $a_1 = a_2$ , as did May<sup>8)</sup> and Smith, <sup>16)</sup> Eq. 9 can be simplified into a following equation;

$$f(I) = \log K_s^{\circ} + CI + DI^2. \tag{10}$$

Table 1 gives the values of  $\log K_i^c$ , C, and D obtained by the least squares method for Eq. 10 in the whole range of ionic strengths (Case 1). Moreover, as being obvious from Fig. 1, there are good linear relationships between f(I) and the ionic strength in the range of low ionic strengths below ca. 10, i.e.,

$$f(I) = \log K_s^{\circ} + CI. \tag{11}$$

Table 1 also gives the values of  $\log K_s^{\circ}$  and C for Eq.

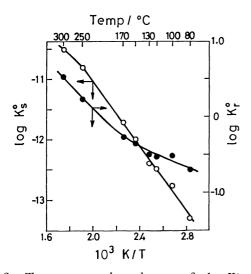


Fig. 2. Temperature dependences of  $\log K_s^{\circ}$  and  $\log K_r^{\circ}$  from 80 to 300 °C for the Case 1.

11 (Case 2).

Figures 2 and 3 show variations of  $\log K_*^\circ$  with the temperature in the case of 1 and 2, respectively. We can observe linear relationships between  $\log K_*^\circ$  and 1/T in the range of temperature from 80 to 250 °C, which are given by following equations:

$$\log K_s^{\circ} = -2663/T - 5.71$$
 (Case 1), (12)

$$\log K_s^{\circ} = -2764/T - 5.45$$
 (Case 2). (13)

According to Eq. 12 or 13, the value of  $\log K_s^\circ$  at 25 °C becomes -14.65 or -14.73, being a little larger than that for gibbsite, -15.18.1) From the values of  $\log K_s^\circ$  at 25 °C for gibbsite and boehmite, the standard Gibbs free energy of following reactions can be calculated as follows:

$$Al(OH)_3 + H_2O = Al(OH)_4^- + H^+,$$
  
 $\Delta G_{299}^{\circ} = 86.61 \text{ kJ},$  (14)

AlOOH + 
$$2H_2O = Al(OH)^{-}_4 + H^{+}_5$$
  
 $\Delta G_{298}^{\circ} = 83.82 \pm 0.23 \text{ kJ}.$  (15)

From the above two equations,

$$Al(OH)_3 = AlOOH + H_2O,$$
  
 $\Delta G_{398}^{\circ} = 2.79 \pm 0.23 \text{ kJ}.$  (16)

This means that gibbsite is more thermodynamically

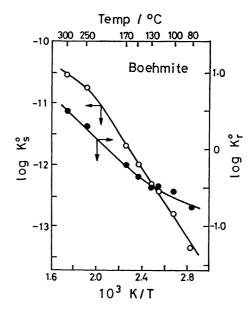


Fig. 3. Temperature dependences of  $\log K_{\rm s}^{\circ}$  and  $\log K_{\rm r}^{\circ}$  from 80 to 300 °C for the Case 2.

Table 1. Temperature dependences of  $K_s^{\circ}$ ,  $K_r^{\circ}$ , C, and D

No.	Temp °C	Case 1				Case 2		
		$\log \widetilde{K_{\mathfrak{s}}^o}$	$C \times 10^2$	$D \times 10^3$	$\log K_{\rm r}^{o}$	$\log \widetilde{K_{\mathfrak{s}}^{\circ}}$	$C \times 10^2$	$\log K_{\mathrm{r}}^{\circ}$
1	80	-13.30	1.32	5.61	-0.71	-13.35	4.78	-0.76
2	100	-12.76	1.43	3.24	-0.52	-12.79	3.35	-0.54
3	120	-12.49	8.34	-5.35	-0.53	-12.44	4.99	-0.48
4	130	-12.34	8.41	-0.05	-0.50	-12.34	8.47	-0.50
5	150	-11.99	7.81	0.00	-0.36	-11.99	7.82	-0.36
6	170	-11.73	9.23	-1.19	-0.27	-11.68	7.27	-0.22
7	250	-10.80	6.50	-0.81	0.24	-10.75	4.95	0.30
8	300	10.51	4.53	-0.12	0.53	-10.53	4.98	0.51

stable than boehmite at 25 °C and 1 atm. From a comparison of the temperature dependences of  $K_s^\circ$  for gibbsite and boehmite, a transition temperature between the two alumina hydrates was found to be  $60\pm3$  °C. Based on the  $\Delta G_s^\circ$  value of  $-915.0~\mathrm{kJ/mol}$  mol selected as a standard Gibbs free energy of formation for boehmite by Parks,<sup>5)</sup>  $\Delta G_s^\circ$  value for Al-(OH)<sup>-4</sup> ion was obtained to be  $-1305.6~\mathrm{kJ/mol}$ , which agrees well with the value of  $-1302.5~\mathrm{kJ/mol}$  obtained in the case of gibbsite. A heat of Reaction 1 is equal to be  $51.9\pm0.8~\mathrm{kJ}$  between 80 and 250 °C, using Eqs. 12 and 13.

For a reaction of boehmite with OH- ion,

$$AlOOH(s) + OH^{-}(1) + H_2O(1) = Al(OH)_{4}^{-}(1)$$
 (17)

$$K_{\rm r}^{\circ} = a_{\rm A1}/a_{\rm OH}a_{\rm w} \tag{18}$$

using Eqs. 2 and 4, K; is given by

$$K_{\rm r}^{\rm o} = K_{\rm s}^{\rm o}/K_{\rm w}^{\rm o}. \tag{19}$$

Table 1, and Figs. 2 and 3 also show the values of  $\log K_{\cdot}^{\circ}$  at various temperatures. They can be expressed approximately as a function of the temperature as follows;

$$\log K_r^{\circ} = 2217/T + 17.32 \log T - 51.10$$
 (Case 1), (20)

$$\log K_r^{\circ} = 1066/T + 11.58 \log T - 33.24$$
 (Case 2). (21)

It should be noted that the values of C, which usually reflect interactions between ions and solvent, have the magnitude of nearly 0.05 commonly for boehmite and gibbsite<sup>1)</sup> in a wide temperature range.

The Case of Different Ionic Size Parameters. In order to evaluate an effect of difference in the ionic size parameters,  $a_1$  and  $a_2$ , on magnitudes of  $K_s^\circ$ , we assumed conveniently  $c_2=0$  and  $d_2=0$  for the dissociation of water as well as did in the previous paper.<sup>1)</sup> Then, from Eqs. 4 and 8,

$$\log (K_{\rm w}/a_{\rm w}) = \log K_{\rm w}^{\rm o} + 2A_{\rm t}I^{1/2}/(1 + B_{\rm t}a_{\rm 2}I^{1/2})$$
 (22)

using  $K_s = m_H m_{A1} = K_w (m_{A1}/m_{OH})$  and Eq. 22,

$$\log (K_{\rm s}/a_{\rm w}^2) = \log (K_{\rm w}^o/a_{\rm w}) + \log (m_{\rm AI}/m_{\rm OH}) + 2A_{\rm t}I^{1/2}/(1 + B_{\rm t}a_2I^{1/2}).$$
 (23)

At a given temperature,  $K_{\rm w}^{\circ}$ ,  $A_{\rm t}$ , and  $B_{\rm t}$  are constant, and so the values of  $\log{(K_{\rm s}/a_{\rm w}^2)}$  depend upon  $(m_{\rm AI}/m_{\rm OH})$ ,  $a_{\rm w}$ ,  $a_{\rm 2}$ , and I. Figure 4 gives plots of  $\log{(K_{\rm s}/a_{\rm w}^2)}$  and  $I^{1/2}/(1+1.59I^{1/2})$  at 150 °C with  $a_{\rm 1}=4.5$  Å and various values of  $a_{\rm 2}$  from 3.5 to 6.0 Å. It is obvious from Fig. 4 that the values of  $\log{(K_{\rm s}/a_{\rm w}^2)}$  and  $\log{K_{\rm s}^{\circ}}$  increase with decrease in the  $a_{\rm 2}$  value.

On the other hand, from Eqs. 2 and 8,

$$\log (K_{\rm s}/a_{\rm w}^2) = \log K_{\rm s}^{\rm o} + 2A_{\rm t}I^{1/2}/(1 + B_{\rm t}a_{\rm l}I^{1/2}) + 2c_{\rm l}I + 2d_{\rm l}I^2.$$
(24)

Figure 4 also shows that the curves approach gradually the theoretical slope,  $2A_t=1.38$ , as the ionic strength decreases, and deviate positively from the straight line in the range of high ionic strength owing to the last two terms in Eq. 24. The similar tendency was observed at different temperatures up to 300 °C, for example, as shown in Fig. 5. Furthermore, such behaviors are consistent with those of gibbsite in NaOH solutions. Therefore, plots of  $\log (K_s/a_*^2)$  vs.  $I^{1/2}/(1 + B_t a_1 I^{1/2})$  consist of the straight line in low I range and the positive deviation in high I range, and the

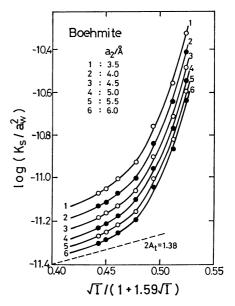


Fig. 4. Plots of  $\log(K_{\rm s}/a_{\rm w}^2)$  vs.  $I^{1/2}/(1+1.59I^{1/2})$  at various values of  $a_2$  in the case of  $a_1=4.5$  Å and 150 °C.

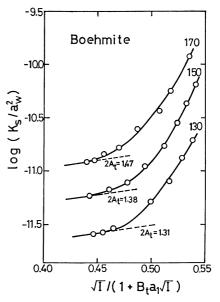


Fig. 5. Plots of  $\log(K_{\rm s}/a_{\rm w}^2)$  vs.  $I^{1/2}/(1+B_{\rm t}a_1I^{1/2})$  at temperatures of 130, 150, and 170 °C in the case of  $a_1=4.5$  Å and  $a_2=5.0$  Å. A value of  $B_{\rm t}a_1$  is equal to 1.57, 1.59, and 1.61 at 130, 150, and 170 °C, respectively.

latter may be attributed to the formation of complex ion(s) such as  $\mathrm{Al_2O(OH)_6^{2-}}$  and/or the dehydration of  $\mathrm{Al(OH)_4^{-}}$  to  $\mathrm{AlO(OH)_2^{-}}$  and  $\mathrm{AlO_2^{-}}$  ions.<sup>12)</sup> It should be noted that the sodium aluminate and hydroxide solutions show the same behavior according to the extended Debye-Hückel expression in the wide range of temperatures from 40 to 300 °C for the both solid phases of gibbsite and boehmite.

## References

1) Thermodynamics of Al<sub>2</sub>O<sub>3</sub>-Na<sub>2</sub>O-H<sub>2</sub>O System. Part 3. Part 2: B.-T. Chang, Bull. Chem. Soc. Jpn., **54**, 1960

(1981).

- 2) A. S. Russell, J. D. Edwards, and C. S. Taylor, J. Metals, **7**, 1123 (1955).
- 3) D. D. MacDonald and P. Butler, Corros. Sci., 13, 259 (1973).
- 4) J. A. Kittrick, Soil Sci. Soc. Am. J., **30**, 595 (1966); **44**, 139 (1980).
  - 5) G. A. Parks, Am. Mineral., 57, 1163 (1972).
- 6) B. S. Hemingway and R. A. Robie, Geochim. Cosmochim. Acta, 41, 1402 (1977).
- 7) B. S. Hemingway, R. A. Robie, and J. A. Kittrick, Geochim. Cosmochim. Acta, 42, 1533 (1978).
- 8) H. M. May, P. A. Helmke, and M. L. Jackson, Geochim. Cosmochim. Acta, 43, 861 (1979).
- 9) V. A. Bernshtein and E. A. Machenok, *Zhur. Prikl. Khim.*, **34**, 982 (1961).

- 10) J. W. Cobble, Science, 152, 1479 (1966).
- 11) T. Okabe, A. Okuwaki, and M.-C. Shieh, *Denki Kagaku*, **39**, 258, 340 (1971).
- 12) I. A. Dibrov, G. Z. Mal'tsev, and V. P. Mashovets, *Zhur. Prikl. Khim.*, **37**, 1920 (1964).
- 13) G. S. Kell, G. E. McLaurin, and E. Whalley, *J. Chem. Phys.*, **48**, 3805 (1968).
- 14) H. S. Harned and B. B. Owen, "The Physical Chemistry of Electrolytic Solutions," 3rd ed, Reinhold, New York (1958), p. 645.
- 15) J. R. Fisher and H. L. Barnes, J. Phys. Chem., **76**, 90 (1972).
- 16) R. M. Smith and J. D. Hem, U. S. Geol. Surv. Water Supply Paper 1827-D (1972).
- 17) R. J. Moolaar, J. C. Evans, and C. D. McKeever, J. Phys. Chem., **74**, 3629 (1970).